Swapping parentheses   P37366_en
Cinquè Concurs de Programació de la UPC - Final (2007-10-03)

Let $\mathcal{P}_n$ be the set of words with exactly n opening parentheses and n closing parentheses, such that every ')' matches a '('. For instance,

$$\mathcal{P}_3 \;=\; \{\ \text{"((()))"}\ ,\ \text{"(()())"}\ ,\ \text{"(())()"}\ ,\ \text{"()(())"}\ ,\ \text{"()()()"}\ \}\ .$$

Consider the following experiment: Choose one word w from $\mathcal{P}_n$ at random. Then, pick one '(' and one ')' of w, independently at random, and swap them. What is the probability that the result is also a word in $\mathcal{P}_n$?

For example, let $n = 3$. If we choose $w =$ "((()))", then there are exactly four swaps that produce a word in $\mathcal{P}_3$, namely 2-4, 2-5, 3-4, 3-5. The rest of swaps (1-4, 1-5, 1-6, 2-6, 3-6) are incorrect. Each of the other words in $\mathcal{P}_3$ has three correct swaps. Therefore, the probability for $n = 3$ is

$$\frac{1}{5}\left(\frac{4}{9} + \frac{3}{9} + \frac{3}{9} + \frac{3}{9} + \frac{3}{9}\right) = \frac{16}{45} \simeq 0.355556\ .$$

Input

Input consists of several integer numbers n between 1 and 30.

Output

For every given n, print with six digits after the decimal point the probability that swapping a random '(' with a random ')' of a random word in $\mathcal{P}_n$ produces a word also in $\mathcal{P}_n$.

Sample input

```
1
2
3
10
30
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Sample output

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0.000000
0.250000
0.355556
0.585699
0.731991
```

Problem information

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