Binomial coefficients P93625_en

Concurso On-line 3 (OIE08) (2008)

The binomial coefficient or choose function $\binom{n}{k}$ is the number of ways to choose k objects from n objects. Its formula is well known:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \ ,$$

where $n! = n \cdot (n-1) \cdots 2 \cdot 1$. This formula is not very useful from a computational point of view, because we have to deal with huge numbers (the factorial numbers) to obtain much smaller results. For instance,

$$\binom{20}{10} = \frac{20!}{10!10!} = \frac{2432902008176640000}{1316819440000} = 184756 \ .$$

Despite the fact that the final number has only 6 digits, we need to compute 20!, which has 19 digits. This can be a problem because the type `int` of 32 bits cannot store numbers with more than 10 digits.

However, this is not the only way to compute $\binom{n}{k}$. For instance, binomial coefficients satisfy the following property:

$$\binom{n}{k} = \begin{cases} 1 & \text{if } k = 0 \text{ or } k = n \\ \binom{n-1}{k-1} + \binom{n-1}{k} & \text{if } 0 < k < n \end{cases}$$

This recursive formula allow us to compute binomial coefficients with no multiplications nor divisions, by using a procedure known nowadays as "Pascal's triangle" or "Tartaglia's triangle", although it has historical references more than 1000 years old:

$$\begin{array}{ccccccccccccccccccccc}
 & & & & & & 1 & & & & & & & & \binom{0}{0} & & & & & & \\
 & & & & & 1 & & 1 & & & & & \binom{1}{0} & & \binom{1}{1} & & & & & \\
 & & & & 1 & & 2 & & 1 & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & \\
 & & 1 & & 4 & & 6 & & 1 & & 1 & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} \\
 & & & & & & \cdots & & & & & & & \cdots & & & & & &
\end{array}$$

To compute more binomial coefficients, you only have to fill more rows of the triangle. Use this idea to compute the value of several binomial coefficients.

Input

Input consists of several cases, each with two natural numbers n and k, where $0 \leq n \leq 30$ and $0 \leq k \leq n$.

Output

For each case, print $\binom{n}{k}$.

Sample input 1

```
0 0
1 0
1 1
2 0
2 1
2 2
```

Sample output 1

```
1
1
1
1
2
1
```

Sample input 2

```
20 10
30 15
30 10
30 20
30 0
30 30
```

Sample output 2

```
184756
155117520
30045015
30045015
1
1
```

Problem information

Author : Omer Giménez
Translator : Carlos Molina
Generation : 2024-05-03 09:12:53