30 Quantum gates

23 points

Introduction

Quantum computing is the new paradigm of the computation, it is said to be the next technology revolution! It takes advantage of the ability of subatomic particles to exist in more than one state at any time. Due to the way the tiniest of particles behave, operations can be done much more quickly and use less energy than classical computers.

In classical computing, a bit is a single piece of information that can exist in two states: 1 or 0. Quantum computing uses quantum bits, aka qubits. These qubits have also two states, but these states can exist simultaneously because of the superposition of these values. This is the magic of quantum computing, a qubit can have the two values at the same time until it is measured!

This behavior is described in terms of the probability that when the qubit is measured becomes 0 or 1. It is expressed through bra-ket notation that basically means to put everything inside | and >, so the 0 now is |0>, the 1 became |1> and the qubit q is notated as |q>. So,

$$|q >= a|0 > +b|1 >$$

means that the probability to be |0> is a2 and the probability to be 1 is b2 where the coefficients a and b are complex numbers. Usually qubits are also expressed following as a vector:

$$|q >= \begin{vmatrix} a \\ b \end{vmatrix}$$

As an example, to describe a qubit with the same probability (50%) to become a 0 or 1 then the value of the coefficients a and b should be:

$$a^2 = \frac{1}{2} \rightarrow a = \sqrt{\frac{1}{2}}$$

$$b^2 = \frac{1}{2} \rightarrow b = \sqrt{\frac{1}{2}}$$

And the qubit will be:

$$|q >= \frac{1}{\sqrt{2}}|0 > +\frac{1}{\sqrt{2}}|1 >$$

As in classical computing, there are logical gates (AND, OR, NOT, ...) to define the circuits; in quantum computing they are known as quantum gates and are used to create quantum circuits. Single-qubit gates are the simplest and these are the Pauli (X, Y and Z) and the Hadamard (H) gates. Since a qubit can be thought of like an imaginary sphere. Whereas a classical bit can be in two states – at either of the two poles of the sphere – a qubit can be any point on the sphere. Consequently, the quantum gates would manipulate the qubit direction inside the sphere. To support such transformations the quantum gates are represented using matrices.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

But despite all this complexity, to solve a quantum circuit is as easy as doing operations with matrix and vectors, so if we represent the qubits as vectors, to solve a quantum circuit is to multiply a matrix with a vector.

Then the circuit, given the input qubit |q> and the quantum gates G1, G2, G3..., GN in line the resulting qubit |q'> will be doing the following operations:

- |q1> = G1 * |q>
- |q2> = G2 * |q1>
- |q3> = G3 * |q2>
- ...
- |q'> = GN * |q(n-1)>

We ask you to program an algorithm that given an input qubit and a quantum circuit using the quantum gates described before, it outputs the resulting qubit.

Input

The input is described in five lines. The first two lines describe the coefficient a, the first line represents the real part of coefficient and the second is the complex (i) part of coefficient. The third and fourth line describe the coefficient b, the third line represents the real part of coefficient and the fourth is the complex (i) part of coefficient. Finally, the last line represents the quantum circuit as a string with the sequence of gates

Output

Write output in terms of (a1 + a2i) |0> + (b1 + b2i) |1> with three decimals precision.

Example 1

Considering the qubit |q> = 1/√2+0i|0> + 1/√2+0i|1> and this quantum circuit: a H gate following by an X gate.

Input

```
0.707
0
0.707
0
HX
```

Output

```
(0.0+0.0i)|0> + (1.0+0.0i)|1>
```

Example 2

Considering the qubit |q> = 1+0i|0> + 0+0i|1> and this quantum circuit: just an X gate.

Input

```
1
0
0
0
X
```

Output

```
(0.0+0.0i)|0> + (1.0+0.0i)|1>
```

Example 3

Considering this qubit |q> = 1.006|0> + 0+0.111i|1> and this quantum circuit: a Z gate followed by a H gate.

Input

```
1.006
0
0
0.111
ZH
```

Output

```
(0.711-0.078i)|0> + (0.711+0.078i)|1>
```

Example 1 developed step by step

Given the qubit (0.707+0.0i)|0> + (0.707+0.0i)|1> and the circuit HX find out the resulting qubit. First step means to apply the gate H to the current qubit:

$$|q1>= H \times |q>= \frac{1}{\sqrt{2}} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

For the second step just apply the gate X to the qubit |q1>:

$$|q2>= X \times |q1>= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So, the resulting qubit is (0.0+0.0i)|0> + (0.1+0.0i)|1>

Matrix operations

Multiplying a real value (A) and a matrix:

$$A \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} A \times a & A \times b \\ A \times c & A \times d \end{bmatrix}$$

Multiplying a matrix and a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} A \times a + B \times b \\ A \times c + B \times d \end{bmatrix}$$

Basic operations with complex numbers

Given two complex numbers z1 and z2,

- z1 = x1 + (y1)i
- z2 = x2 + (y2)i

then the basic operations are:

- Adding two complex numbers: z1 + z2 = (x1 + x2) + (y1 + y2)i
- Subtracting two complex numbers: z1 - z2 = (x1 - x2) + (y1 - y2)i
- Multiplying a complex number z1 by a real number A: A * z1 = A * x1 + (A * y1)i
- Multiplying two complex numbers: z1 * z2 = (x1 * x2) - (y1 * y2) + (x1 * y2 + x2 * y1)i